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RESPONSE OF IMPACT DAMPERS WITH GRANULAR MATERIALS UNDER RANDOM EXCITATION

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SUMMARY

This paper presents the results of an experimental and analytical study of the performance of granular material dampers with tungsten powder, as an impacting mass, under wide-band random excitation. The influence of some of the major system parameters such as the total auxiliary mass ratio, container dimensions and intensity of the excitation are investigated using a small building model under base excitation. An approximate analytical solution based on the concept of an equivalent single-unit impact damper is presented. Comparison between the experimental and analytical results shows that, with the proper use of the equivalent single-particle impact damper approach, reasonably accurate estimates of the rms response of a primary system under stationary random excitation can be obtained.

KEY WORDS: damping; non-linear; experimental; granular; impact; tungsten

1. INTRODUCTION

1.1. Background

Impact dampers are a class of highly non-linear auxiliary mass dampers which employ plastic deformations and momentum transfer between an impacting mass, which moves freely within the boundaries of a container, and the vibrating system to which the damper is attached, in order to attenuate the oscillations of the primary system. An idealized model of a single-particle impact damper is shown in Figure 1.

Impact dampers were initially used to limit undesirable oscillations more than half a century ago. In the intervening years, numerous analytical and experimental studies of these dampers have been conducted, and many practical applications have been reported. Representative publications concerning this class of dampers include the work of Araki *et al.*,[1] Bailey and Semercigil,[2] Bapat and Sankar,[3,4] Cempel,[5] Cempel and Lotz,[6] Chen and Semercigil,[7] Cronin and Van,[8] Grubin,[9] Hales,[10] Jing and Young,[11] Kato *et al.*,[12] Lieber and Jensen,[13] Mansour,[14] Masri and Caughey,[15] Masri,[16–25] Masri and Ibrahim,[26,27] Masri and Yang,[28] Nigm and Shabana,[29] Panossian,[30] Papalou,[31] Papalou and Masri,[32] Popplewell and Bapat,[33] Popplewell and Semercigil,[34] Popplewell and Liao,[35] Rocke and Masri,[36] Roy *et al.*,[37] Semercigil and Popplewell,[38] Semercigil *et al.*,[39] J. Shaw and W. S. Shaw,[40] Shaw *et al.*,[41] Skipor and Bain,[42] Sung and Yu,[43] Thomas and Sadek,[44] Thomas *et al.*,[45] Veluswami *et al.*,[46] Warburton,[47] Welt and Modi,[48,49] Yasuda and Toyoda,[50] and Ying and Semercigil.[51]

Among the desirable features of impact dampers for structural control, are their ruggedness, reliability and simplicity to design. Furthermore, studies have shown that this class of dampers is capable of having superior performance to conventional linear auxiliary mass dampers in regard to controlling the response of structures subjected to earthquake-like disturbances.[28] However, the significant impact forces generated during the use of single-particle impact dampers may result in large peak accelerations, high noise levels and

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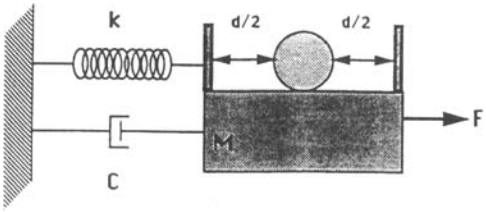


Figure 1. A single-degree-of-freedom system with an attached impact damper

rapid deterioration of surface materials involved in the contact process. In addition, changes in the container size and the excitation amplitude may affect considerably the performance of a single-particle impact damper. These facts have motivated the investigation of multi-unit dampers and multi-particle single-unit dampers (i.e. situations where the particles are operating in parallel or series) constrained to operate in a uni-axial fashion, as well as the general case of multi-particles in a single container which allows a 'sloshing'-type of three-dimensional motion. Among the significant advantages of such forms of multi-particle impact dampers is: (1) the substantial reduction of the peak level of impulsive forces exchanged during momentum transfer, (2) the attendant reduction in the accompanying plastic deformations induced in the damping process and (3) the appreciable attenuation of the noise level surrounding the operation of the damping device.

In spite of the numerous analytical and experimental studies that have been conducted over the years into the various aspects of the motion of the impact damper, many questions remain unanswered regarding the physics of the very complicated non-linear phenomena involved in the operation of multi-particle impact dampers of general shape and size under arbitrary dynamic excitations. Not only does this class of dampers involve some fascinating features of chaotic oscillations related to 'hardening' non-linearities, but it also involves dissipative internal friction forces, and inelastic multi-body impact problems in multidimensions. Furthermore, depending on many variables such as the particle-container geometry, packing density, and level and direction of excitation, the particle-damper system exhibits phenomenologically different regimes of motion where the dominant interaction forces between the primary system and the damper can change drastically. In view of the above formidable challenges to the theoretical studies of particle dampers, it is not surprising that there is a paucity of analytical results, even approximate ones, for determining the response of structural systems provided with particle dampers under dynamic loads.

1.2. Scope

The authors have recently conducted a comprehensive experimental investigation of multi-particle impact dampers under a variety of dynamic loads and with different types (discrete particles, granular materials), sizes and number of particles, and with different combinations of key system parameters.[32] The focus of the study was to gain further insight into the nature of the highly non-linear dynamic interaction forces between the primary system and the particles, with particular attention devoted to identifying the optimum combination of system parameters leading to the most efficient (from the response reduction point of view) damper design parameters, when using damper mass ratios that are relatively small.

The focus of this paper is on the performance of a particle damper with a representative granular material (tungsten powder), as an impacting mass, under wide-band random excitation. The influence of the following system parameters is investigated: total auxiliary mass ratio, container dimensions and intensity of the excitation. An approximate analytical solution, based on the generalization of the experimental results using the concept of an equivalent single-unit impact damper, is presented for estimating the primary system rms levels when operating in the vicinity of the optimum combination of system parameters. The experimental and analytical results are compared in order to establish the range of validity of the approximate solution and to demonstrate its accuracy.

2. EXPERIMENTAL SET-UP AND PROCEDURE

The experimental model consisted of the primary structure, operating as an equivalent Single-Degree-Of-Freedom (SDOF) system, on which the particle damper was mounted. The structure was made of aluminum, acted as a SDOF system, and was designed to be lightly damped with a natural frequency of approximately 11 Hz. The particle damper consisted of four aluminum edge brackets. One of the brackets was fixed to a plate mounted on the structure; the other three were free to move, thus allowing convenient modification of the damper size. The total mass of the system was 4·90 kg, and the primary system had a damping ratio of $\zeta = 0{\cdot}01$. The particle damper and the experimental set-up are shown in Figure 2. The dimensions of the box-like damper are d_x, d_y, d_z, where d_x is the separation of the walls of the damper that are perpendicular to the direction of the excitation, d_y is the separation of the walls of the damper that are parallel to the direction of the excitation and d_z is the maximum height that the balls can reach.

The system was excited randomly at the base by means of a hydraulic shaker and a random-noise generator, which provided a repeatable sequence of random numbers with a bandwidth of 0–50 Hz. The motion of the structure relative to the base, as well as the base motion, was monitored with accelerometers and direct displacement gauges.

3. SYSTEM RESPONSE

The impacting mass of the particle damper consisted of grains of tungsten powder. The tests were conducted by keeping the level of excitation and the mass ratio μ constant ($\mu = m/M$, where m is the total mass of the particles and M is the mass of the primary structure) while varying the size of the container as shown in Figure 3. Initially, d_y was kept constant and d_x was varied from the largest possible value to the smallest.

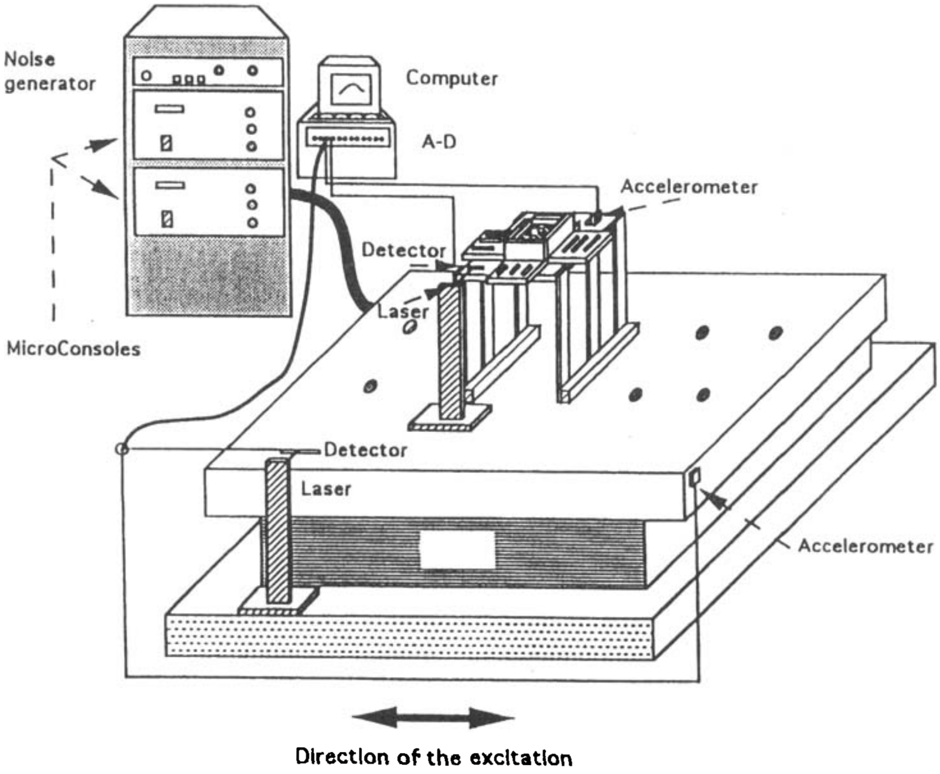


Figure 2. Experimental set-up

Then d_y was decreased, and the same procedure was followed, until the smallest combination of d_x and d_y was obtained. Sample results are presented in Figure 4 for the case of $\mu = 0{\cdot}027$ and $\mu = 0{\cdot}108$ (representing a four-fold increase in the auxiliary mass ratio). The abscissa and ordinate of the graphs have been made dimensionless by dividing the dimensions of the damper d_x, d_y and the rms value of the response displacement x in the presence of the damper, by the rms value of the response displacement σ in the absence of the damper.

3.1. *Effects of container dimensions*

When the mass ratio was $\mu = 0{\cdot}027$ the response curves were smooth, and close to each other (Figure 4(a)). The influence of the container size on the response of the structure is noticeable only when the dimensions of the damper were small. The maximum attenuation of the response amplitude reached 35 per cent.

For larger mass ratios, the influence of the container size on the response of the structure is higher. When the mass ratio was $\mu = 0{\cdot}108$ for small damper sizes, the response amplitude was relatively high and the response curves (Figure 4(b)) were close to each other and converging to a point. The high response amplitudes were due to the fact that, when the container dimensions were small, the grains of tungsten

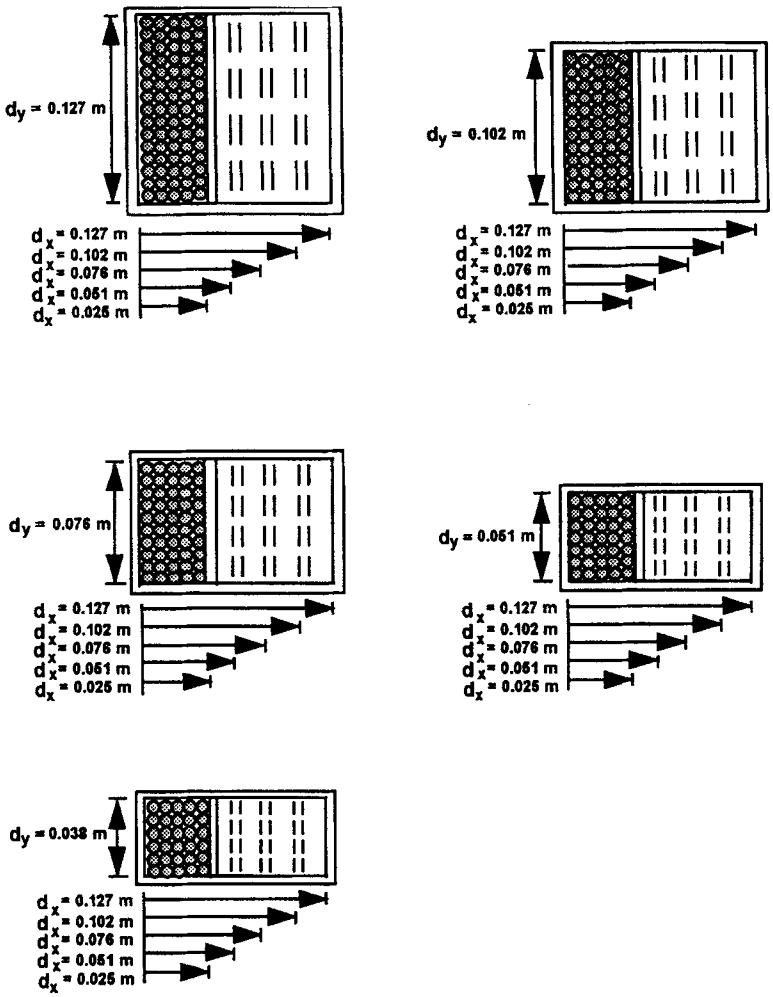


Figure 3. Dimensions of the particle damper used in the experiments

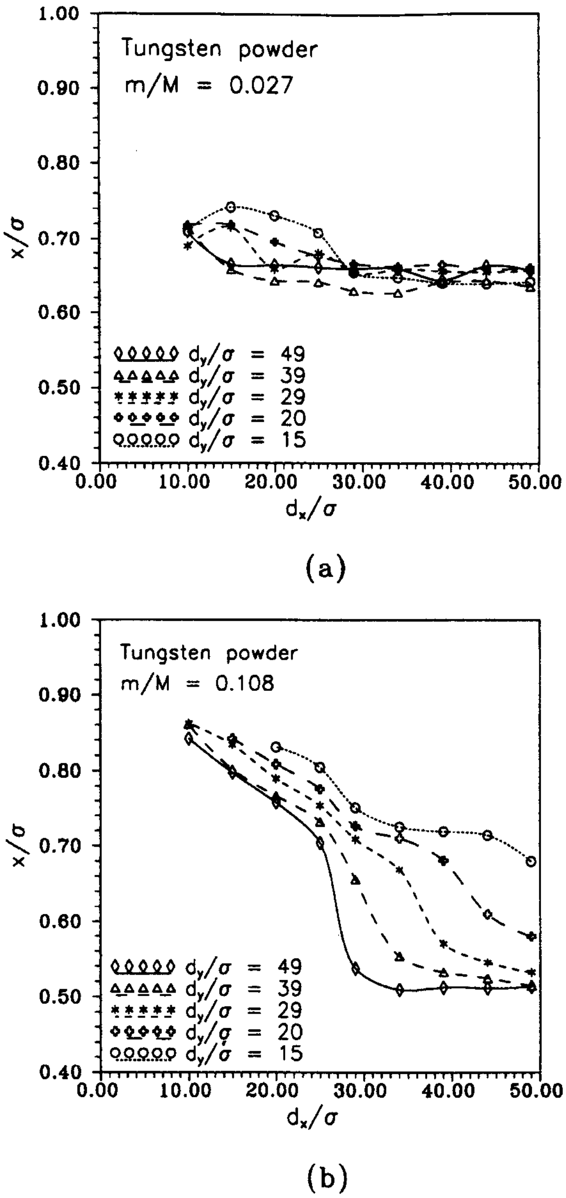


Figure 4. Response of a base-excited SDOF system with an attached granular material damper under random excitation with different mass ratios: (a) $\mu = 0{\cdot}027$ and (b) $\mu = 0{\cdot}108$

powder were gathered in many layers which minimized their motion. The internal friction of the granular material was the dominant mechanism in the interaction between the primary and auxiliary mass, which resulted in a smaller exchange of momentum and a decrease in the effectiveness of the damper. As the size of the damper was increased, momentum transfer between the primary and auxiliary mass became the main interaction mode, thus resulting in a decrease in the response amplitude. The maximum attenuation of the response amplitude reached 50 per cent.

3.2. *Effects of mass ratio*

Three different values of μ were considered: 0·027, 0·054 and 0·108 (a ratio of 1 : 2 : 4). The results are presented in Figure 5 for $d_y/\sigma = 49$. For relatively large sizes ($d_y/\sigma > 30$), the increase of the mass ratio decreases the

response amplitude in non-linear fashion. As the size of the damper decreases, the response amplitude (for most cases) increases with the increase of the mass ratio. This phenomenon occurs because the larger the mass ratio, the more grains of tungsten powder were piled one above the other reducing their mobility (increase in internal friction). The reduction of motion results in smaller exchange of momentum and higher amplitudes. Higher excitation could increase their motion and the corresponding exchange of momentum.

3.3. *Effects of excitation level*

The influence of the excitation level on the performance of the particle damper was examined by keeping the mass ratio constant at $m/M = 0{\cdot}108$ and using four levels of excitation. For each of the different levels of excitation, the dimension d_y was kept constant and d_x was varied. Figure 6 shows that as the level of

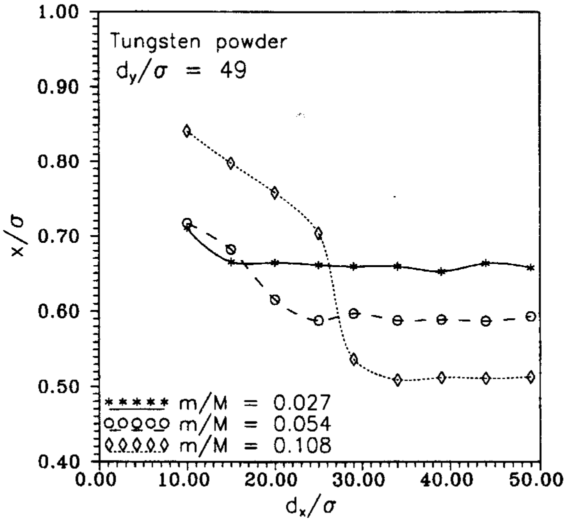


Figure 5. Influence of the mass ratio on the performance of a granular material damper attached on a base excited SDOF system under random excitation, with $d_y/\sigma = 49$

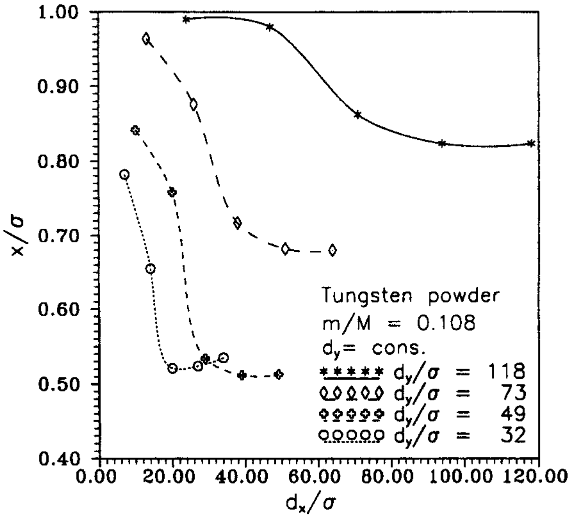


Figure 6. Influence of the random excitation level on the performance of a particle damper attached to a base-excited SDOF system, with $m/M = 0{\cdot}108$

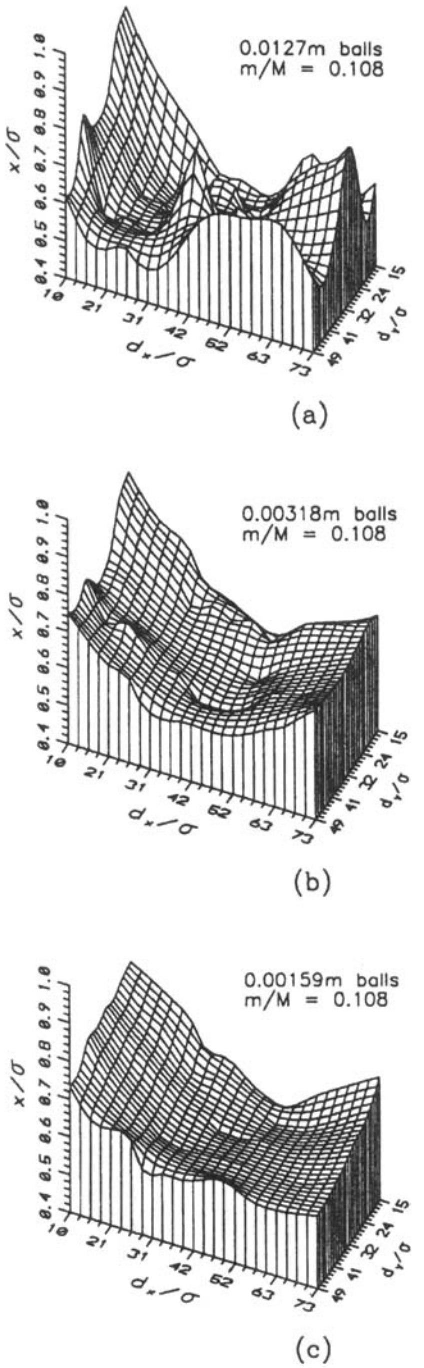


Figure 7. Three-dimensional representation of the response of a base-excited SDOF system with an attached particle damper under random excitation, with mass ratio of 0·108; diameter of the particles: (a) 0·0127 m, (b) 0·00318 m and (c) 0·00159 m

excitation decreases (resulting in an increase in the dimensionless gap size in the direction orthogonal to the excitation axis) the response amplitude increases. However, when the excitation amplitude is high enough to overcome the friction forces between the particles, the response becomes independent of the intensity of the excitation provided the dimensionless clearance ratio remains constant.

3.4. *Influence of damper material*

In addition to the granular material under discussion, the present experimental study dealt with particle dampers employing different materials (ball bearings) of various sizes and total mass. Detailed, experimentally derived, quantitative response curves of the response of the primary system when provided with various types of particle dampers are available in the work of Papalou.[31] Sample results showing the response of the experimental model under discussion, when provided with a particle damper employing ball-bearings of different sizes, are shown in Figure 7.

As one would expect, the results indicate that as the diameter of the discrete particles becomes progressively smaller, the behaviour of the damper with particles approaches that of granular materials such as tungsten powder. However, due to increased internal friction in granular materials such as tungsten powder, as compared to the spherical balls used in the aforementioned tests, the response of the primary system in the vicinity of the optimum range of system parameters becomes less sensitive to variations in the clearance size.

3.5. *Further implementation issues*

In addition to the influence of the above-listed parameters on the system response, additional issues arise in the practical application of granular materials for vibration control under earthquake-like disturbances, where the multicomponents of the excitation cause more complicated interaction effects among the damping materials. Also, depending on the packing density of the granular materials, the arrangement and settling of the granular materials may have a significant impact on the transient response of the system. The issues require further research.

4. APPROXIMATE ANALYTICAL SOLUTION

The results of the experiments discussed above can be used to design optimally a multi-particle damper for a structure of mass M that is excited randomly. The precise analysis of a dynamic system with an attached particle damper of the type under discussion is quite complex; consequently, an approximate approach is used to generalize the results. Since the theory for a single-particle damper has already been developed, one approach is to find a method to replace the granular material damper with an equivalent single-particle damper. This is accomplished by replacing the individual particles of the particle damper with a single particle having an equivalent mass, coefficient of restitution, and clearance.

4.1. *Equivalent approach*

There are several alternative ways to define the equivalent single-particle damper for a multi-particle unit. The approach followed here is to equate the empty volume in a single-particle damper with the empty volume in a multi-particle damper, as depicted in Figure 8. A particle damper from a given material (tungsten in this case) is uniquely characterized by its dimensions d_x, d_y, and the total mass of the particles m.

The volume that the grains of tungsten powder occupy in a multi-particle damper, V_{spd}, is:

$$V_{\mathrm{spd}} = \frac{m}{\rho_{\mathrm{tun}}} \tag{1}$$

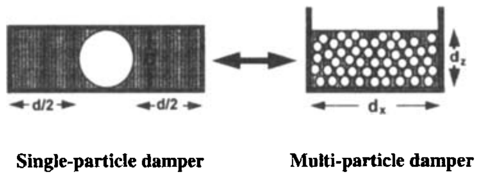
where ρ_{tun} is the density of the grains of tungsten powder.

The empty volume left in a multi-particle damper, V_{epd}, is

$$V_{\mathrm{epd}} = V_{\mathrm{pd}} - V_{\mathrm{spd}} = d_x d_y d_z - \frac{m}{\rho_{\mathrm{tun}}} \tag{2}$$

where V_{pd} is the volume of the particle damper, a box-shaped container in the present study.

Hence, given the dimensions of the particle damper d_x and d_y, and the total mass of the particles m, the only unknown in equation (2) is the dimension d_z. Our experimental measurements of d_z, suggested that a good approximation of the packing density[10] $\rho_{\mathrm{p}} = V_{\mathrm{spd}}/V_{\mathrm{pd}}$ is $\rho_{\mathrm{p}} = 0{\cdot}34$. Substituting the above numerical



$$V_{eid} = \frac{\pi D^3}{12} + \frac{\pi}{4} D^2 d \equiv V_{epd} = d_x d_y d_z - \frac{m}{\rho_{tun}}$$

$$\frac{\pi D^3}{12} + \frac{\pi}{4} D^2 d = 1.94\, V_{spd}$$

Figure 8. Empty-volume equivalence of a single-particle impact damper and a granular material damper

values into equation (2), the empty volume of the multi-particle damper can be expressed in the following form:

$$V_{epd} = 1{\cdot}94 V_{spd}. \tag{3}$$

The empty volume of a single-particle cylindrical damper, V_{eid}, is given by

$$V_{eid} = \frac{\pi}{4}\delta^2 h - \frac{1}{6}\pi D^3 \tag{4}$$

where D is the diameter of the particle in the single-particle damper, δ and h are the diameter and length of the cylindrical single-particle damper, respectively.

Assuming $D \simeq \delta$, and since $h = D + d$ (d is the clearance of the cylindrical single-particle damper), then equation (4) can be written in the form

$$V_{eid} = \frac{\pi D^3}{12} + \frac{\pi}{4} D^2 d. \tag{5}$$

According to the present approach, the equivalent single- and multi-particle dampers have equal empty volumes. Consequently, equating (3) and (5) leads to

$$1{\cdot}94 V_{spd} = \frac{\pi D^3}{12} + \frac{\pi}{4} D^2 d \tag{6}$$

Since the mass m^* of the particle in the single-particle damper can be obtained from

$$m^* = \rho\left(\pi \frac{D^3}{6}\right),$$

where ρ is the density of the particle, the diameter D in equation (6) can be replaced by

$$D = \left(6\frac{m^*}{\pi\rho}\right)^{1/3}. \tag{7}$$

Substitution of equations (1) and (7) into equation (6) results in

$$\frac{m}{\rho_{tun}} = 0{\cdot}26\frac{m^*}{\rho} + 0{\cdot}62 d\left(\frac{m^*}{\rho}\right)^{2/3}. \tag{8}$$

4.2. Equivalent damper

From the analytical studies that have been done for a single-particle damper under random excitation, if one knows the mass m^*, the clearance d, and the coefficient of restitution e, the rms response of the structure x/σ can be determined analytically.[26] The dimensionless rms response curves of the base-excited single-degree-of-freedom system with an attached single-particle damper under random excitation are shown in Figure 9.

The procedure for determining an approximate solution corresponding to an equivalent single-particle impact damper, utilizes the results of the previous section and the following steps:

1. Given a primary system of mass M and damping ζ that is provided with a particle (granular material) damper of specified dimension (d_x, d_y) whose particles have a density ρ_{tun}, mass ratio $\mu = m/M$, and an equivalent coefficient of restitution e. Imposing the condition that this particle-damper empty volume must match the corresponding empty volume in a single-particle damper, leads to equation (8) which is of the form
$$d = f(\mu_1) \tag{9}$$
where $\mu_1 = m^*/M$.
2. By determining the response of a single-particle impact damper under random excitation, having the same ζ and e of the particle-damper material, a parametric set of curves of the form
$$\left(\frac{x}{\sigma}\right) = g\left(\frac{d}{\sigma}, \mu_1\right) \tag{10}$$
can be generated[21] for various combinations of (d/σ) and μ_1, as shown in Figure 9.

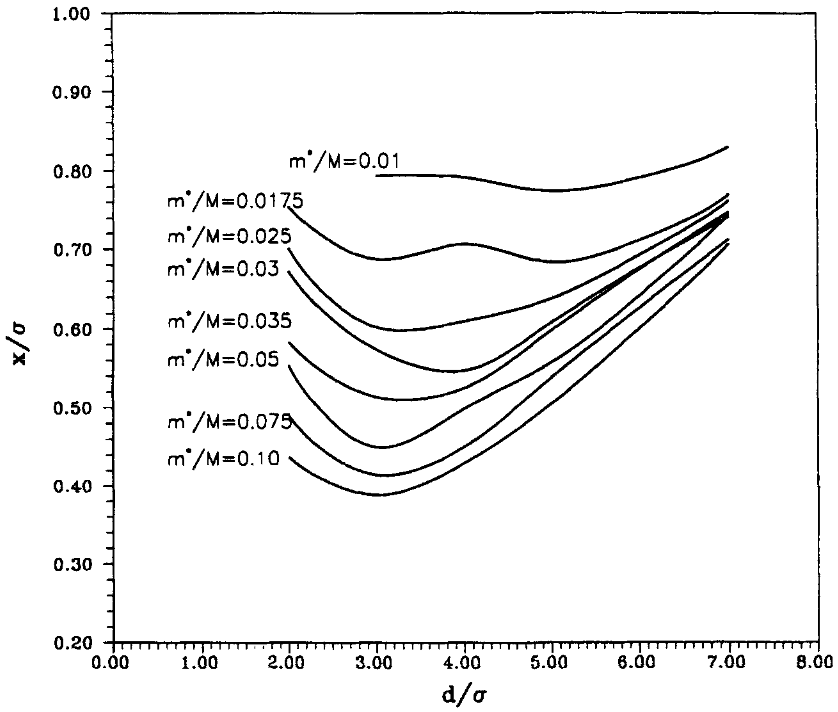


Figure 9. Response of a base-excited SDOF system with an attached single-particle impact damper under random excitation for $e = 0{\cdot}25$, $\zeta = 0{\cdot}01$ and $m^*/M = 0{\cdot}01$, $0{\cdot}0175$, $0{\cdot}025$, $0{\cdot}03$, $0{\cdot}035$, $0{\cdot}05$, $0{\cdot}075$, and $0{\cdot}10$

3. Using standard iteration techniques for the numerical solution of systems of non-linear algebraic equations, equations (9) and (10) can be solved to yield the value of μ_1 for which the dimensionless response ratio (x/σ) of a single-particle damper matches the corresponding ratio achieved by the particle damper specified in step 1.
4. By repeating the above steps for different d_x/σ ratios, a single curve of the form shown in Figure 10 is obtained. Similar procedures can be used for different mass ratios.

The above procedure was used for all the experimental results acquired when the mass ratio $\mu = m/M$ was equal to 0·027, 0·054 and 0·0108. For optimum design purposes, representative curves of the equivalence of a single-particle damper to a multi-particle damper were obtained and are shown in Figure 10. Since Figure 4 shows that the optimum design is accomplished when

$$29 \lesssim \frac{d_y}{\sigma} \lesssim 49,$$

thus d_y/σ must be restricted to lie within this range in order to achieve optimum performance.

4.3. Optimum design of particle damper

The steps that one can follow to design an optimum particle damper, with granular material such as tungsten powder as an impacting mass, attached to a lightly damped single-degree-of-freedom primary system are summarized below.

Given the mass of the primary structure M, the total mass of the granular material particles m, and the random excitation intensity level.

(a) The dimension d_y must satisfy the following condition:

$$29 \lesssim \frac{d_y}{\sigma} \lesssim 49.$$

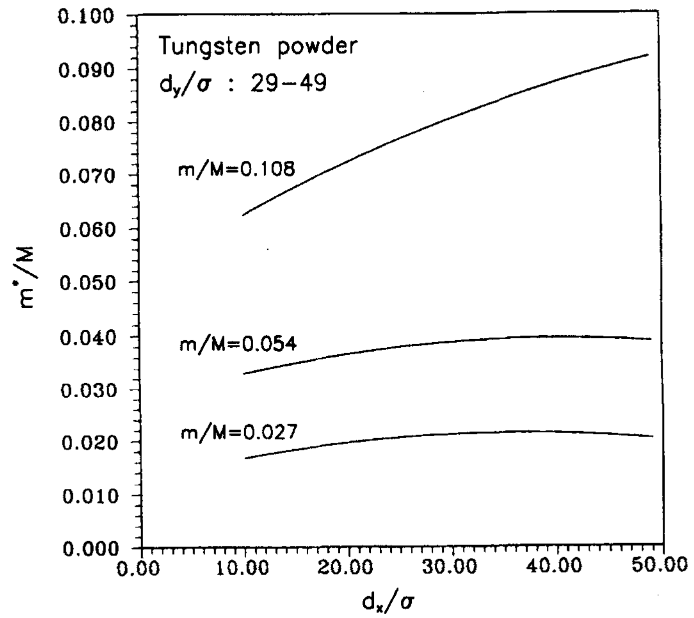


Figure 10. Representative curves of the equivalence of a single-particle impact damper to a granular material damper employing tungsten powder

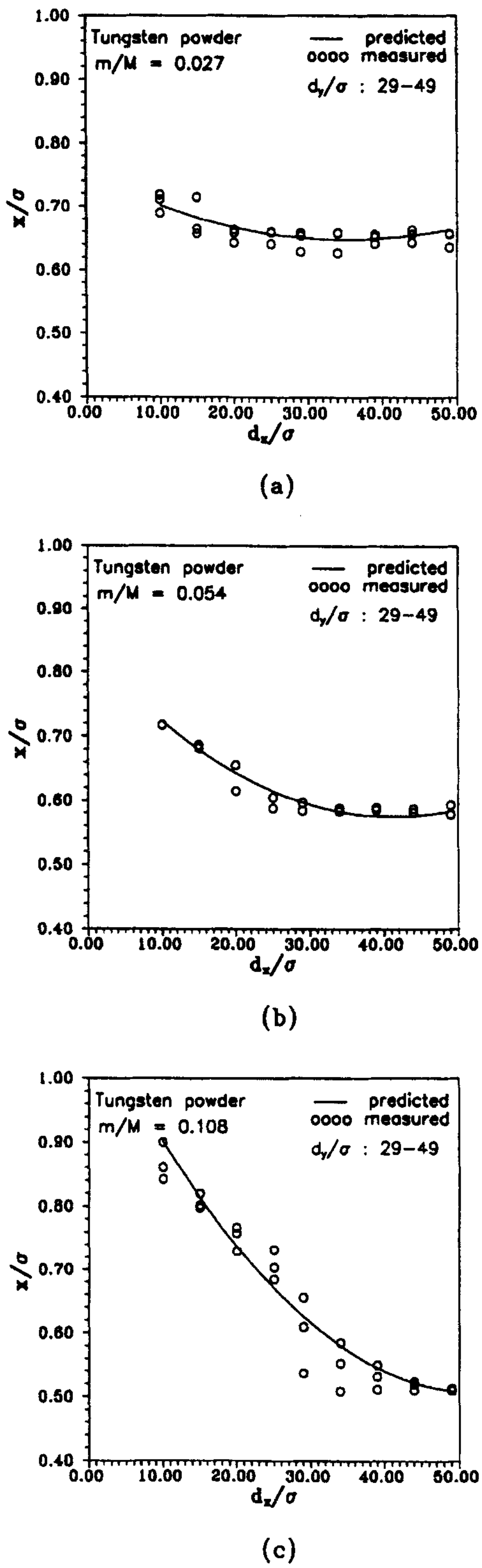


Figure 11. Comparison of measured and predicted results for an SDOF system provided with a granular material damper under random excitation

(b) According to Figure 4, the optimum design can be achieved if d_x is within the range

$$29 \lesssim \frac{d_x}{\sigma} \lesssim 49.$$

Therefore, a good first selection of d_x is a value that satisfies the above inequality.

(c) From Figure 10, the mass ratio m^*/M of an equivalent single-particle damper can be found.

(d) Substituting the values of m, ρ and m^* into equation (8) yields the clearance d of an equivalent single-particle damper.

(e) Using Figure 9, the response of the structure with the particle damper can be estimated.

If the response of the structure is not satisfactory, alternate values for d_x and d_y can be selected and the above procedure repeated.

4.4. Validation of approximate solution

The inverse of the scheme used above can be employed to calibrate the accuracy of the proposed equivalence method.

From the representative curves of Figure 10, and for each d_x/σ and m/M, a corresponding m^*/M is found. Using equation (8) to find d, and from the curves in Figure 9 the response of the structure is obtained. Figure 11 shows the predicted and measured values.

Clearly, for the range of system parameters used in the experimental phase of this study, the proposed equivalence approach to utilize the performance characteristics of a single-particle damper to estimate those of a corresponding granular material damper utilizing a material such as tungsten powder, yields reasonably accurate results over a broad range of parameters that span the optimum range of damping efficiency.

5. CONCLUSIONS

The performance of a particle damper, with tungsten powder as an impactor, used for controlling the forced vibrations of structures was investigated under random excitation. The influence of some of the key system parameters (such as total auxiliary mass ratio, container dimensions and intensity of the excitation) on the behaviour of the particle damper are considered.

It is found that, by replacing the solid particle in the container of a single-particle damper by grains of tungsten powder (or similar granular material) of equal mass, significant improvements are achieved with respect to reduction of high noise levels, interface material deterioration, and reduction in the sensititivity of the particle-damper effectiveness to the size of the container and to the excitation amplitude. Also, higher values of the mass ratio result in a proportionally non-linear (i.e. less efficient per unit mass ratio) decrease of the response amplitude.

By invoking the concept of an equivalent single-particle damper, an approximate analytical solution is provided to estimate, with reasonable accuracy, the response of a randomly excited primary structure which is equipped with a particle damper (with tungsten powder) operating in the vicinity of the optimum range of its system parameters.

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APPENDIX

Nomenclature and definition of symbols

x root-mean-square value of the primary system displacement in the presence of the particle damper

σ root-mean-square value of the primary system displacement in the absence of the particle damper

e	damper coefficient of restitution
ζ	ratio of critical damping of the primary system
d_x	separation of the walls of the damper that are perpendicular to the direction of the excitation
d_y	separation of the walls of the damper that are parallel to the direction of the excitation
d_z	maximum height that the balls can reach (depth of particle container)
M	mass of the primary structure
m	total mass of the particles in the multi-particle damper
m^*	mass of the particle in the single-particle damper
μ	mass ratio, m/M
μ_1	mass ratio, m^*/M
ρ	particle density in the single-particle damper
ρ_{tun}	density of the grains of the tungsten powder
ρ_p	packing density
D	diameter of the particle in the single-particle damper
V_{pd}	volume of the particle damper
V_{spd}	volume that the particles occupy in a multi-particle damper
V_{epd}	empty volume left in a multi-particle damper
V_{eid}	empty volume of a single-particle cylindrical damper
δ	diameter of the cylindrical single-particle damper
h	length of the cylindrical single-particle damper
d	clearance of the cylindrical single-particle damper

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